Indian Statistical Institute
Second Semester Examination 2004-2005
M.Math II Year
Graph Theory and Combinatorics

Time: 3 hrs Date:17-05-05 Max. Marks : 100

1. Define a 2-design. Show that any non-trivial 2-design has at least as many blocks as points. Characterize the case of equality. [20]

2. Let D be a 3-(22, 6, 1) design. Show that there exist numbers $x < y$ such that any two distinct blocks of D have either x or y points in common. Consider the graph G whose vertices are the blocks of D; two (distinct) vertices are adjacent iff the corresponding blocks have exactly x points in common. Calculate the parameters of G as a strongly regular graph. [30]

3. Let H be a strongly regular graph with parameters (16, 5, 0, 2). Show that for any vertex x of H, the induced subgraph of H on the non-neighbours of x is a strongly regular graph K. Calculate the parameters of K. [25]

4. Let $A = ((a_{ij}))$ be a real matrix of size $n \times n$. Show that

$$\det(A) \leq \sqrt{\prod_{i=1}^{n} \sum_{j=1}^{n} |a_{ij}|^2}.$$

If, further $|a_{ij}| = 1$ for all i, j and equality holds in the above inequality then show that A is a Hadamard matrix and $n = 1, 2$ or $n = 0 \pmod 4$. [25]